

Theorem [I]

If  $a, b, c \in G$ , then prove that

$$(i) ab = ac \Rightarrow b = c \quad (\text{left cancellation law})$$

$$(ii) ba = ca \Rightarrow b = c \quad (\text{right cancellation law})$$

OR,

State and prove cancellation laws in a group.

Proof: (i) Here, we have to prove that

$$ab = ac \Rightarrow b = c \quad \forall a, b, c \in G \quad (\text{left cancellation law})$$

For this, suppose  $ab = ac$ , then we shall try to show that  $b = c$ .

Let  $a^{-1}$  be the inverse of  $a$  in  $G$ .

Multiplying both sides of (i) by  $a^{-1}$  on the left, we get

$$a^{-1}(ab) = a^{-1}(ac)$$

$$\Rightarrow (a^{-1}a)b = (a^{-1}a)c, \text{ by associative law}$$

$$\Rightarrow eb = ec, \text{ since } a^{-1}a = e, \text{ the identity in } G.$$

$$\Rightarrow b = c, \because e \text{ is the identity}, \therefore eb = b, ec = c.$$

Hence first part of the theorem is proved.

Hence the proof (i).

Proof (ii): We have to prove that  $ba = ca \Rightarrow b = c$ .

For this, suppose  $ba = ca$ , then to show that  $b = c$ .

Let  $a^{-1}$  be the inverse of  $a$  in  $G$ . Multiplying both sides of (ii) by  $a^{-1}$  on the right, we have

$$(ba)a^{-1} = (ca)a^{-1}$$

$$\Rightarrow b(aa^{-1}) = c(aa^{-1}), \text{ by associativity}$$

$$\Rightarrow be = ce, \because aa^{-1} = e, \text{ the identity in } G.$$

$$\Rightarrow b = c, \text{ since } e \text{ is the identity}, \therefore be = b, ce = c.$$

Hence the proof.

re. right cancellation law is proved